**Path Integral Formulation of GF**

So say we have some Hamiltonian,



And we want to evaluate the time ordered GF [going to presume fermions, but bosons can be done by basically taking spins off]:



where |GS> is the interacting ground state. Then following manipulations analogous to the ones made for the single particle. Basically, to write it as a path integral, we must relate to a bosonic/fermionic coherent state. And we use the supposition that in the long time limit, we can neglect overlap between any state other than the GS, because the others will have much higher oscillation frequency. So we start with two arbitrary field states |ψσ,a> and |ψσ,b>. Recall that |ψσ> = |ψ↑>|ψ↓> is an eigenstate of the two operators σ(x) for all x (and σ).



which allows us to conclude,



Using these identities, we can manipulate it into the form:



And inserting resolutions of identity into this expression, we can massage it into the following form, as was done for the propagator (probably should see that file in this folder). I guess I’ll repeat that analysis. So consider:



where ψb and ψa are many-body coherent states. This is the denominator of our GCσσ´(x,t;x´,t´) guy, FWIW. It doesn’t look like it will be necessary to distinguish between bosons and fermions in what follows. We’ll just default to Fermions I guess, but if bosons then ignore the spin sum part. Say the Hamiltonian is:



Normally we insert resolutions of identity that diagonalize the operators – the position/momentum operator. This time we’ll fill in the bosonic/fermionic coherent state resolution of identity. Since we have a spin up and spin down operator, our identity resolution will be:



where we use the last expression as shorthand for the former,



This will give us, generalizing our result above to allow for multiple spin indices:



And now we’ll group the overlap terms together, and the H’s together.



Now to first order in δt, it seems we can change some of the ψ’s. For instance, change ψσ,b\* → ψσ,n-1\* in the top line, ψσ,n-1\* → ψσ,n-2\* in the second line, etc. And in the last line, I’m going to do the same thing. Since the error is first order in δt, that makes the overall error in the exponent (δt)2, and therefore negligible in the small δt limit.



and now we can introduce derivatives,



Taking the continuum limit, we can write this as a functional integral, and we have:



where the integration bounds are to indicate that we have the boundary conditions ψσ(x,-ta) = ψσ,a(x) and ψσ(x,tb) = ψσ,b(x). And



The combination of derivatives is just Im[ψ\*∂tψ]. But an integration by parts on say the first term, will make this:



It stands to reason that likewise, we may write:



Now we explicitly take the ta,b 🡪 ∞ limit, and recognize that apparently in this limit, the identity of the |ψσ,a(**x**)>, |ψσ,b(**x**)> is pushed to irrelevance, we get just get:



where,

